

USING TABLE SAW METHODS TO MAKE OBJECTS HAVING POLYGONAL SHAPES

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1. [Introduction](#)

Since we are going to be dealing with objects that in some way involve polygonal shapes, perhaps it would be best to start by saying what a polygon is because that might be a faded memory from distant grade-school days. A n -sided polygon is a closed object obtained by connecting n straight-line segments, with each line touching the other lines at only two points. In Figure 1: (a) is a five-sided polygon; (b) is a six-sided polygon; (c)

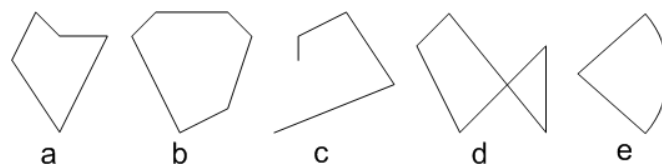


Figure 1. Polygon definition

is not a polygon because it is not closed; (d) is not a polygon because there are lines touching other lines at more than two points; and, (e) is not a polygon because all its lines are not straight. Here is some terminology used with polygons:

<i>edge</i>	...	a line that forms a side of the polygon
	...	a point where two edges meet
<i>vertex</i>	...	the angle, inside the polygon, between two edges that meet at a vertex
<i>interior angle</i>	...	a polygon in which all edges have the same length and all interior angles are equal
<i>regular polygon</i>	...	the angle subtended by an edge of a regular polygon at the center of the polygon
<i>central angle</i>	...	shorthand for a regular polygon with n edges
<i>n-gon</i>		

Several n -gons and their numbers of edges and names are displayed in Figure 2.

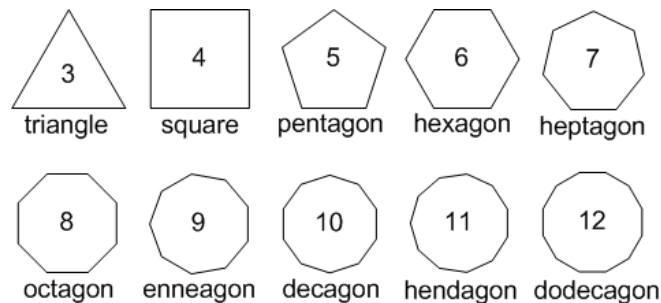
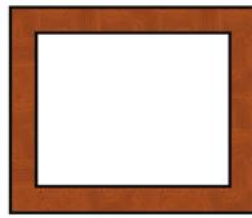


Figure 2. Examples of regular polygons

Regular polygons have the property that they can be surrounded by a circle that touches all vertices, called the *circumscribed circle*. It is the smallest circle that lies entirely outside the polygon. The center of a regular polygon is the center of its circumscribed circle. This implies that the central angle of an n -gon is equal to $360^\circ / n$; for example, each edge of a hexagon subtends an angle of $360^\circ / 6 = 60^\circ$ degrees at its center. The interior angles of an n -gon are the same and equal $180^\circ (n - 2) / n$ degrees; for example, the interior angles of a square are $180^\circ (4 - 2) / 4 = 90^\circ$ degrees and of an octagon are $180^\circ (8 - 2) / 8 = 135^\circ$ degrees. The interior and central angles are supplementary; they sum to 180° , so one of them easily determines the other. A second circle can also be drawn for regular polygons, called the *inscribed circle*. It is the largest circle that lies entirely inside the polygon; it is tangent to all edges and concentric with the circumscribed circle. A regular polygon can have any number of edges greater than three. As the number of edges increase, the shapes of the regular polygons get closer and closer to their circumscribed and inscribed circles, and eventually in the limit they converge to a single circle.



Lidded Box



Picture/Mirror Frame



Picture/Mirror Frame



Cutting Board



Tray

Some objects involving polygonal shapes with no sloping sides are displayed in Figure 3.

Figure 3. Polygonal shapes having no sloping sides: requires simple miter cuts

The top of the lidded box and the bottom of the tray are octagons of solid wood, with the sides of both made from rectangular shapes connected together to form hexagons. The picture/mirror frames are rectangular (not a 4-*gon*) and hexagonal. The rectangular frame and hexagonal cutting board are not regular polygons but are obtained from regular forms by stretching two parallel sides. Making the objects in Figure 3 requires a *simple bevel cut*, as discussed below in Section 2.

Some objects involving polygonal shapes with sloping sides are displayed in Figure 4.



rectangular trays



heptagon vase



hexagon table-lamp

Figure 4. Polygonal shapes having sloping sides: requires compound miter cuts

The vase has sides that slope outward, from bottom to top, 7° from the vertical and a heptagonal (7-*gon*) cross section at each level. The sides of the table lamp slope inward, from bottom to top, 5° from the vertical and a hexagonal (6-*gon*) cross section at each level. Making these requires a *compound bevel cut*, as discussed below in Section 2.

1.1. Polygon Mathematics

Here we review some mathematical aspects of polygons that are useful when making polygonal objects. This might be a major “yuk” or a “ho-hum” for some averse to digging up trigonometry memories, but the knowledge is useful if making polygons is a goal. This is a summary to keep the discussion focused on the facts a woodworker having such a goal needs.

1.1.1. Regular Polygons

The quantities of importance when making an object involving a regular polygonal shape are the interior angles, the length of an edge, and the overall size of the object. Displayed in Figure 5 is a segment of an n -gon with its inscribed and circumscribed circles. A highlighted triangle with three sides (with lengths labeled R , r , and l) and three angles (labeled A_1 , A_2 and A_3 degrees) is shown; its sides are formed by the radii of the circumscribed and inscribed circles and the line between the endpoints of these radii on the circles, one endpoint at the site of tangency with the inscribed circle and the other endpoint at the vertex for the circumscribed circle. The radius of the circumscribed circle is R , which is a measure of the overall size of the n -gon because it lies within a circle of diameter $2R$ and, also, within a square having sides of length $2R$. The angle A_1 is half the central angle and so is equal to $180^\circ / n$ degrees for an n -gon. Angle A_2 is 90° because a radius of a circle meets a tangent to the circle at a right angle. This implies that A_3 is the complement of A_1 , so $A_3 = 90^\circ - A_1 = 90^\circ (n - 2) / n$, which is half the interior angle. Two

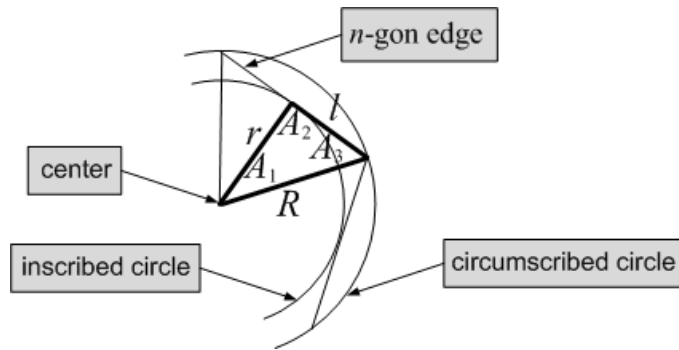


Figure 5. Polygon geometry

sides determine the third side of the triangle because $R^2 = r^2 + l^2$. Sides r and l in terms of the angle A_1 are given by $r = R \cos A_1$ and $l = R \sin A_1$. The length L of an edge is $L = 2l = 2R \sin A_1$, which is another measure of the size of the n -gon.

Example: Consider a pentagon having edges of length $L = 1$ in. Angle $A_1 = 180^\circ / 5 = 36^\circ$ degrees, and $A_2 = 90^\circ - 36^\circ = 54^\circ$. The pentagon lies inside a circle of radius $R = L / (2 \sin A_1) = 1 / (2 \sin 36^\circ) \approx 0.85$ in.

Example: Consider a hexagon to fit inside a circle having a diameter of 1.7 in. The radius of the circumscribed circle is $R = 1.7 / 2 = 0.85$ in. The angles $A_1 = 180^\circ / 6 = 30^\circ$ and $A_3 = 60^\circ$. The length of an edge is $L = 2R \sin A_1 = 1.7 \sin 30^\circ = 0.85$ in.

The angles A_1 and A_3 are important when setting up a table saw to make an object involving the shape of a regular polygon. They are listed in Table 1 for some frequently encountered n -gons: the square, pentagon, hexagon, and octagon.

Table 1. Angles for selected n -gons				
n	central angle	internal angle	A_1	A_3
4	90°	90°	45°	45°
5	72°	108°	36°	54°
6	60°	120°	30°	60°
8	45°	135°	22.5°	67.5°

1.1.2. Irregular Polygons

Irregular polygons – those having edges of unequal lengths – are also of interest. Examples are seen in the cutting board and rectangular picture/mirror frame in Figure 3 and the corner cabinets in Figure 6. Here the angles and edge lengths are less obvious but depend on the particular shape. More can be said for the stretched shape, such as those of the cutting board and picture/mirror frame. Their angles and edge lengths follow in a straightforward way from the underlying regular polygonal shapes from which they are derived.



Figure 6. Corner cabinets having irregular polygonal shapes

2. Miter Terminology

No miter is cut on a table saw when the plane of the saw blade is perpendicular to both the plane of the saw surface and the plane of the miter gauge surface, as in Figure 7(a). The angle indicator of the miter gauge in this position shows 90° . The angle shown by blade-tilt indicators on table saws is the complement of the angle between the plane of the blade and the plane of the surface (i.e., 90° minus the angle between the blade and surface), so the tilt indicator shows 0° in Figure 7(a). A simple miter results when one or the other, but not both, of the blade or miter gauge is angled away from these perpendicular positions, as shown in Figure

7(b) and (c). A compound miter results when both are angled away from their perpendicular positions, as in Figure 7(d).

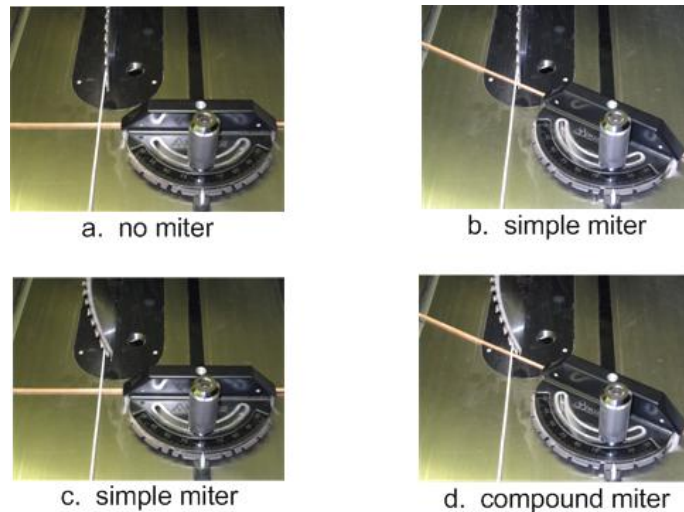


Figure 7. Miter types

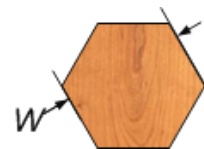
3. Table-Saw Methods for Making Polygonal-Shaped Objects Having Sides Without Slope

Making polygonal shaped objects having sides that do not slope, such as those in Figure 3, requires cutting simple miters. For items like the lid of the box and the bottom of the tray in Figure 3, it is natural to position the blank to be cut so it is flat on the surface of the saw. The blade would then be set perpendicular to the saw's surface (tilt indicator at 0°) and the miter gauge set so that its angle to the saw-kerf is A_3 . For the sides of the box and the tray, with the blanks flat on the surface of the saw, the blade would be set at an angle A_3 to the surface of the saw (tilt indicator at $90^\circ - A_3 = A_1$) and the miter gauge set at an angle perpendicular (i.e., 90°) to the saw kerf.

Example: One approach for making the octagonal top of the lidded box of Figure 3 is to draw the octagon on the blank to identify the cut lines for the eight edges and then use the simple miter setup of Figure 7(c) with the blade perpendicular to the saw surface and the miter gauge set to angle $A_3 = 90^\circ (n - 2) / n = 90^\circ \times 6 / 8 = 67.5^\circ$. An alternative method for making the top is described in Section 3.2 below.

3.1. A Special Method for Cutting Hexagons on a Table Saw

The method we describe for cutting hexagons is adapted from Jim Cummins' [7.1] method for the band-saw. It is safe, accurate and repeatable, so it is suitable when one or multiple identical hexagons are required. The smallest width W of a hexagon is the perpendicular distance between two of its opposing edges, which is the diameter of its inscribed circle; from Figure 5, $W = 2r$. The diameter of the circumscribed circle is $2R = 2r / \cos A_1 = W / \cos 30^\circ \approx 1.16W$. Following is a method that requires an accurate cut of stock to width W , a stop block cut to angle $A_3 = 60^\circ$, and setting a fence accurately to be at angle $A_3 = 60^\circ$ to the saw kerf.



Step 1. (Preliminaries) Cut a length of stock to the desired width W . The length is not critical but should be at least the diameter of the circumscribed circle, $1.16W$, but does not need to be significantly greater than that. Make a jig, for example of $\frac{1}{2}$ in. MDF, in the form shown in Figure 8 to be used as a stop block in Step 5. The size is not critical, but the 60° angle should be accurate.

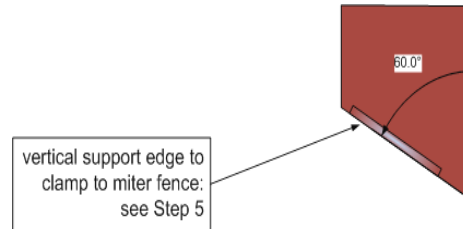


Figure 8. 60° degree jig

Step 2. Set up the table saw and miter gauge as shown Figure 9.

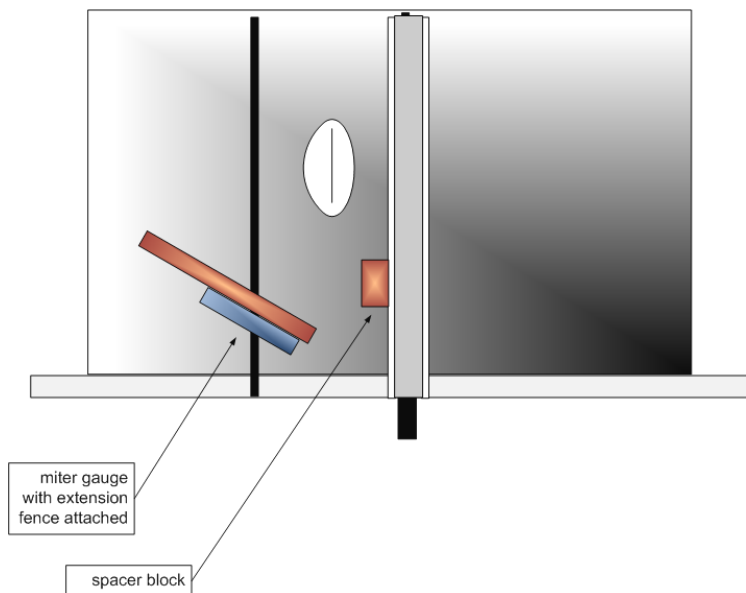


Figure 9. Setup of table saw

In this setup, the miter gauge with attached fence can be replaced by a sliding crosscut box for added accuracy and safety. The purpose of spacer block is to reduce the chance of kick back when making the first two cuts described below; it reduces the chance that cut off pieces can jam between the fence and saw blade and be tossed back towards the user. The miter-gauge angle and fence position are unchanged in completing the first and second cuts.

Step 3. The stock cut to width W and length at least $1.16W$ is positioned for the first cut as shown in Figure 10. Note that the cut line (left side of saw kerf) should pass through the right end of the stock somewhere between the center of the stock and the opposing edge.

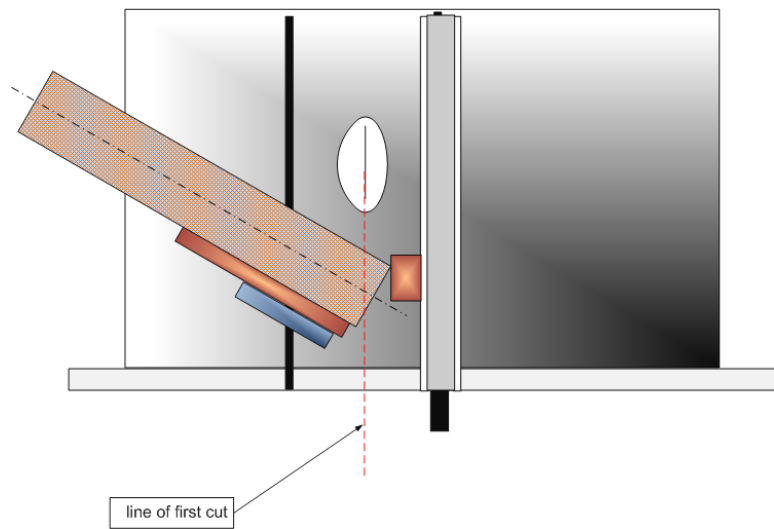


Figure 10. Setup for first cut

The result of the first cut is shown in Figure 11.



Figure 11. Result of first cut

Step 4. Now flip the stock over, and put it in place for the second cut, as shown in Figure 12.

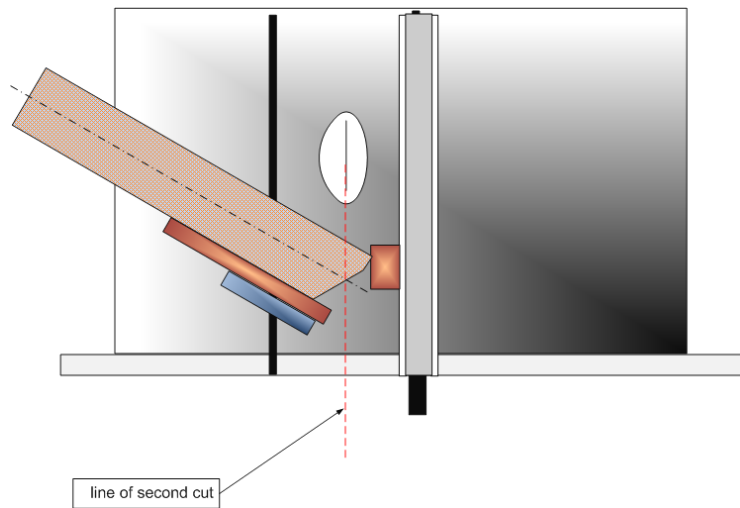


Figure 12. Setup for second cut

The result of the second cut yields the stock with two sides of the hexagon, as in Figure 13.

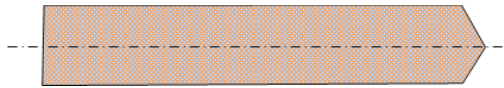


Figure 13. Result of second cut

Step 5. While leaving the miter gauge angle unchanged, place and clamp the 60° stop jig to the miter fence at a distance W from the saw cut line, as in Figure 14. This can be done using the stock since it's width is W , as shown in Figure 15. Move the saw fence away from the saw blade, or remove it altogether, so it does not interfere with the stock on subsequent cuts.

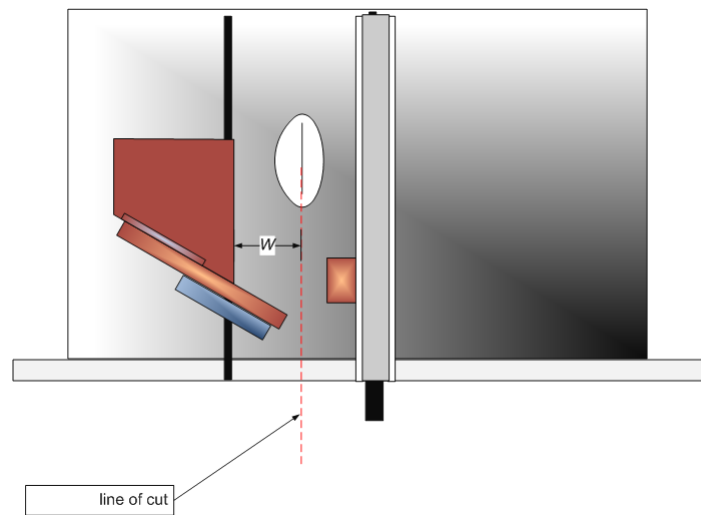


Figure 14. Saw setup for third cut

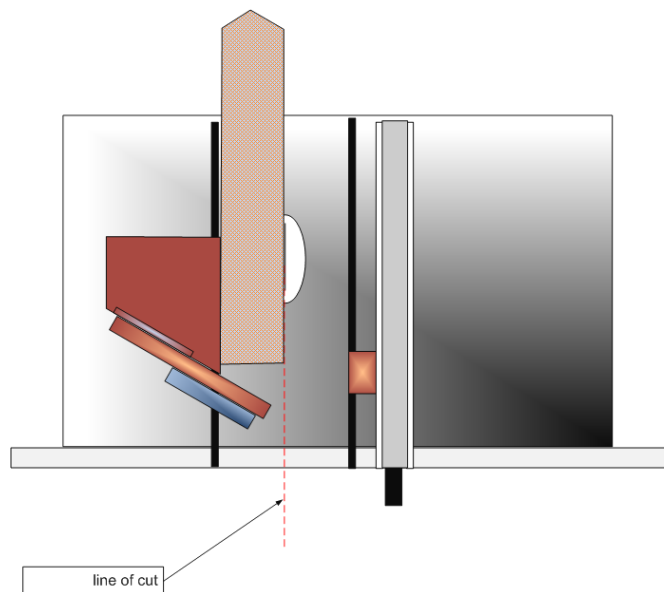


Figure 15. Using The stock to setup for third cut

Step 6. Place the stock as shown in Figure 16, and make the third cut.

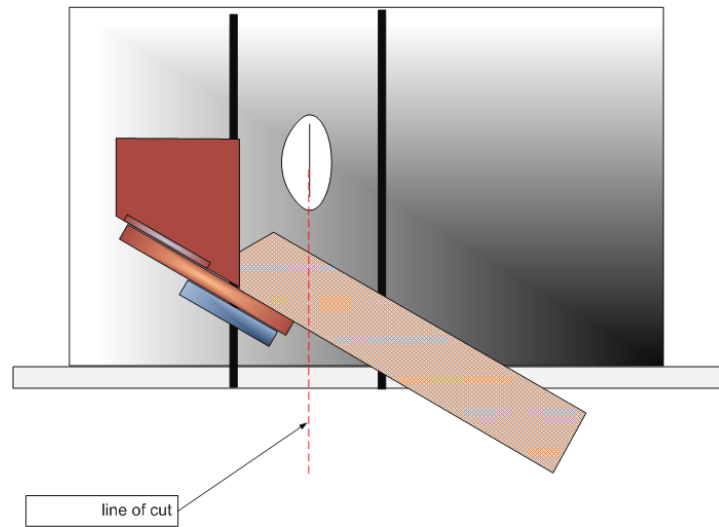


Figure 16. Third cut

The result of the third cut is shown in Figure 17.

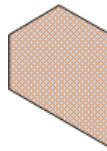


Figure 17. Result of third cut

Step 7. Rotate or flip the result of the third cut, and place it for the fourth, and last cut, as in Figure 18.

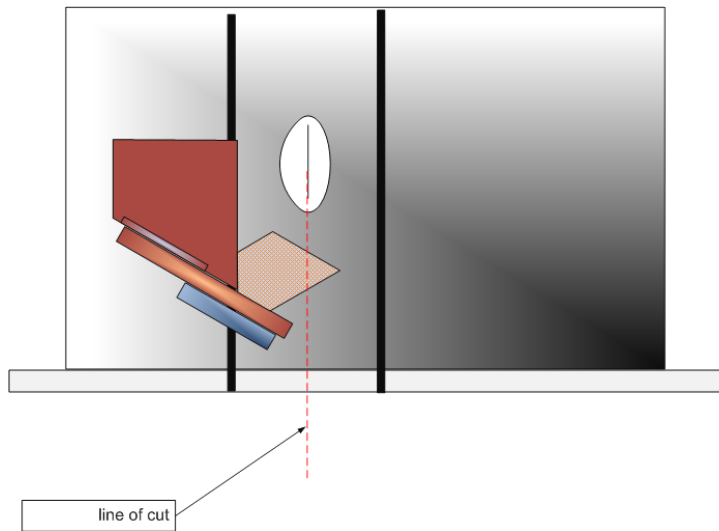


Figure 18. Saw setup for fourth cut

The completed hexagon is shown in Figure 19.

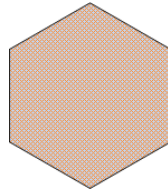


Figure 19. Completed hexagon

3.2. A Pattern-Following Method for Cutting Regular and Irregular Polygons on a Table Saw

Pattern cutting on a table saw can be used for making regular and irregular polygons. It is useful when one or multiple identical copies are needed. The method follows the pattern cutting methods described by Jim Cummins [7.2] and Steve Latta [7.3] and demonstrated in internet videos [7.4,7.5]. The method requires an auxiliary fence that serves to guide a pattern of the polygonal shape to be cut. The auxiliary fence can be clamped to the saw surface but then needs to be carefully adjusted to be parallel to the saw blade (see Cummins [7.2]). It is more easily clamped to the saw fence (see [7.3, 7.4, 7.5]) as in Figure 20. The auxiliary fence is

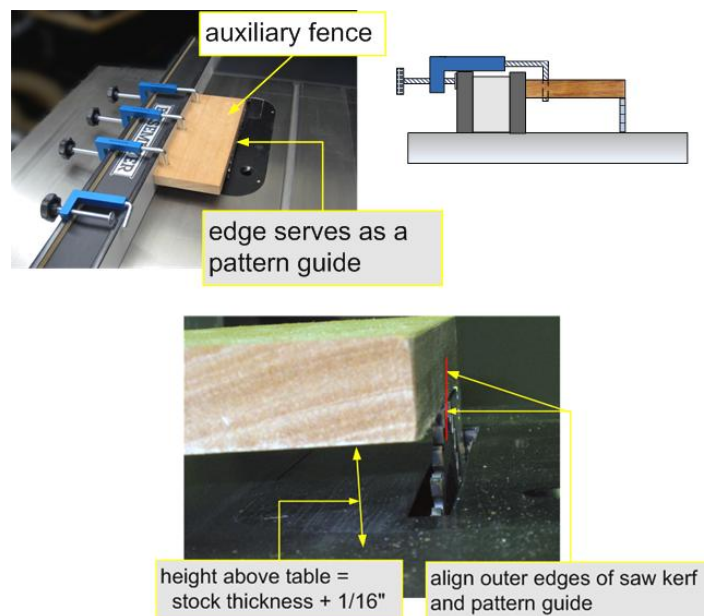


Figure 20. Auxiliary fence serves as a pattern guide

positioned above the saw surface by an amount somewhat greater than the thickness of the wood blank to be cut and clamped firmly in place. The fence is adjusted so the edge of the auxiliary fence that serves to guide the pattern is aligned with the saw kerf. For safety, the saw kerf should not extend beyond the guide edge, but it can be inset from the edge, with the amount of inset taken into account when the pattern is made. The usual recommendation is for the outer edge of the saw kerf to be aligned with the guide edge. The pattern of the polygonal shape to be made is placed on the wood blank in a way that it stays in place throughout the cutting steps. This can be accomplished by gluing sandpaper to the pattern for a friction-only attachment to the blank or by using two-sided tape or screws for a firm attachment.

The blank with pattern attached is passed through the saw using the edge of the auxiliary fence to guide the cut, as indicated in Figure 21. This operation is repeated to cut all sides

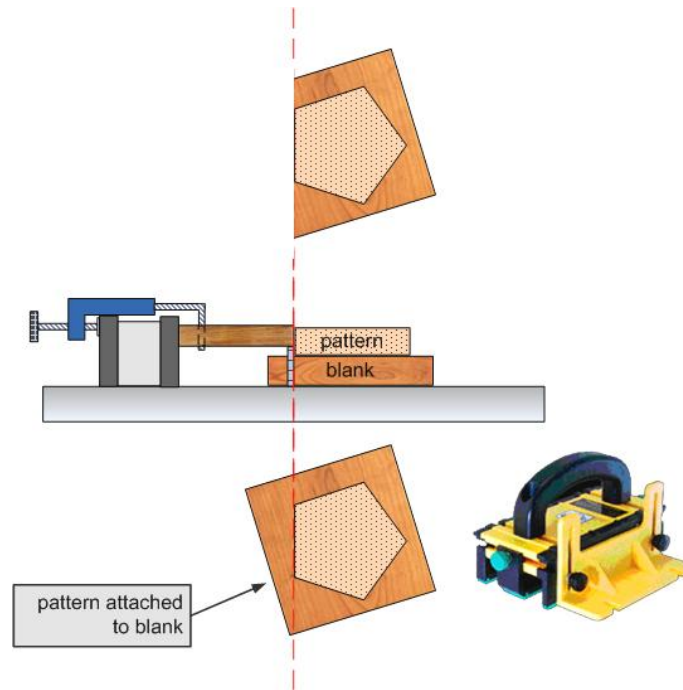


Figure 21. Use of pattern and pattern guide

of the polygonal shape. The pattern with blank should be guided through the cuts in a safe manner using a push stick or other device. Also, for safety, the saw should be turned off and any cutoff material removed from under the auxiliary fence so it does not accumulate there.

An accurate pattern of the polygonal shape to be made is central to this method. There are many ways that the pattern can be made. For example, it can be made by drawing it on the material to be used for the pattern, such as MDF or plywood, and then cutting out the pattern on a table saw or a band saw by carefully following the lines marking the edges or by cutting close to the lines and cleaning up with a hand plane or belt sander. It is difficult to do this accurately, but an adequate result can be achieved with care. Another approach is to use pattern cutting. A scrap board of suitable size with a straight edge is used as a guide by positioning it on the pattern material so its straight edge is aligned with an edge of the polygonal shape. Then, the board fixed to the pattern material is cut using a pattern guide as described above. Cutting each edge in this way yields the pattern. Another approach is computer based. The polygon is drawn with a drawing program, such as Microsoft Visio, and a file created that is suitable for a computerized laser-cutting machine. A plastic pattern can be made inexpensively in this way by an engraving shop. Then the plastic pattern can be used directly for pattern cutting or to make a thicker pattern out of MDF or plywood.

As described up to here, this pattern cutting method can be used to make the lid of the lidded box, the cutting board, and bottom of the tray of the items in Figure 3 and the top and shelves of the corner cabinets in Figure 6 because these cuts do not have bevels. However, simple bevel cuts can also be made. All that is required is to adjust the angle of the saw blade to the desired tilt. It is important for safety to keep all the blade beneath the auxiliary fence, which means that the top of the blade will be inset from the guiding edge of the auxiliary fence. This offset needs to be taken into account when making the pattern to be used if the polygonal shape being made has specified dimensions, but this is a straightforward task with a test cut to measure the offset or a little trigonometric calculation given the tilt angle

and the thickness of the blank. An example for designing the template for making a beveled pentagon is in Figure 22. The desired pentagon has the dimensions of the solid line, the bevel has an angle of

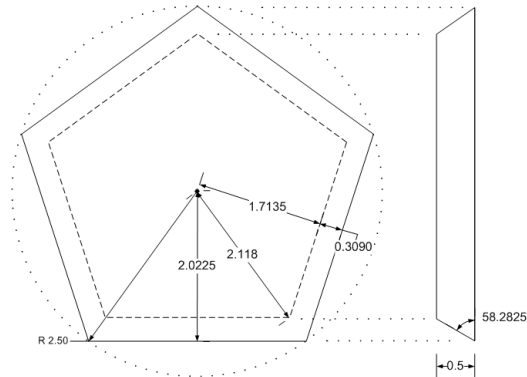


Figure 22. Pattern design for a beveled pentagon

58.2825° , and the pattern has the dimensions of the dashed line. The pattern, of course, is made with no bevel because its edges must glide along the guide face of the auxiliary fence. The setup for making the cuts is shown in Figure 23 after four of the five edges of the beveled

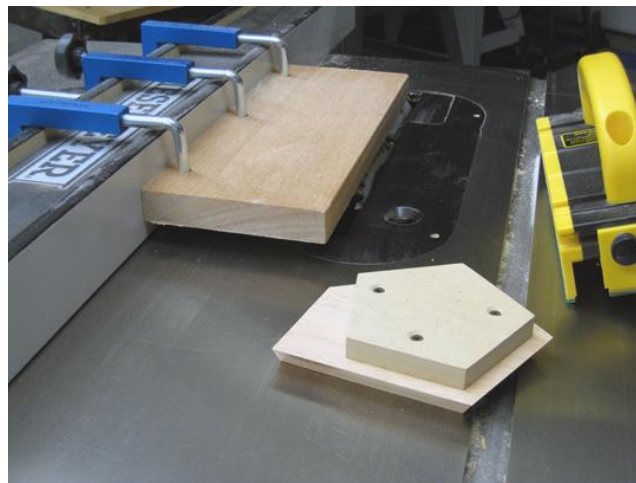


Figure 23. Pattern cutting a beveled pentagon

pentagon have been cut. Because of its small size, it is important for accuracy and safety that the pattern be firmly attached to the blank. Here, this is accomplished with double-sided tape along with three screws whose tips protrude slightly into the face of the mahogany blank. As discussed in Section 5, twelve beveled pentagons made with this pattern cutting method are used to construct a dodecahedron.

4. Polygonal-Shaped Objects With Sides that Slope

Some objects require that compound bevels be cut. Examples are displayed in Figure 4. Following are the trigonometric formulas that govern setting up a table saw to make the cuts. Two angles are needed. One is the angle that the saw blade must make to the surface of the table saw, and the other is the angle that the miter gauge must make with the saw blade (or miter slot if that is parallel with the blade). These angles are defined in the Figure 24.

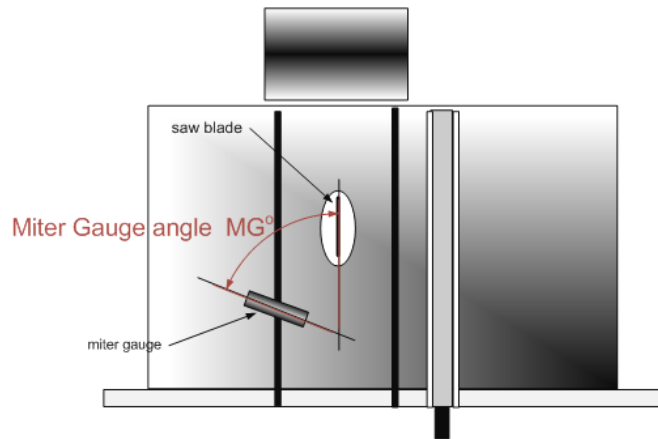
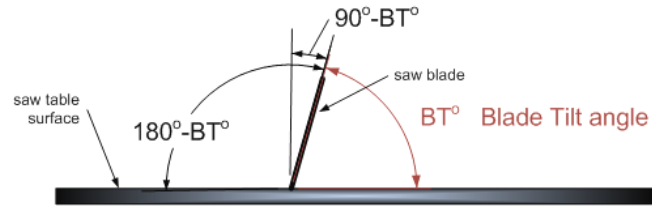


Figure 24. Angles for cutting compound miters

Nomenclature:

BT°	Blade Tilt angle (degrees) measured from table surface example: $BT^\circ = 90^\circ$ is blade vertical
MG°	Miter Gauge angle (degrees) measured from blade surface example: $MG^\circ = 90^\circ$ is cut perpendicular to blade
N	number of sides to container example: $N = 4$ is a rectangular container
S°	slope (degrees) of the container sides measured from base

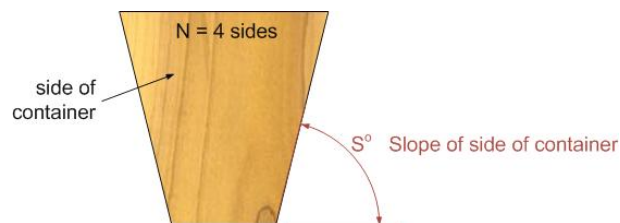


Figure 25. Slope of container sides

Example: $S^\circ = 90^\circ$ is a container with vertical sides

$S^\circ = 80^\circ$ is a container with sides that slope ten degrees from vertical

Notes:

1. Blade tilt indicators on table saws usually show 0° when the plane of the blade is perpendicular to the surface of the saw. When setting the blade tilt using this indicator, use the complement of the blade tilt angle, which is $90^\circ - BT^\circ$, as the indicator angle.
2. When using an adjustable bevel gauge to set the tilt of the saw blade, it may be convenient to set the gauge and use the supplementary angle $180^\circ - BT^\circ$.
3. The drawing above assumes a table saw with a right (clockwise) tilting blade and a miter gauge in the left saw-table slot. These will be reversed for a saw with a left (counterclockwise) tilting blade.

Formulas (see [7.6]-[7.10]):

$$\tan MG^\circ = \frac{1}{\cos(S^\circ) \tan\left(\frac{180^\circ}{N}\right)}, \text{ and } \cos(BT^\circ) = \sin(S^\circ) \sin\left(\frac{180^\circ}{N}\right).$$

Example (rectangular box with vertical sides): For this, $N = 4$ and $S^\circ = 90^\circ$. Then, $\cos(S^\circ) = 0$, $\tan(180^\circ / N) = \tan(45^\circ) = 1$, so $\tan(MG^\circ) = \infty$, implying $MG^\circ = 90^\circ$ degrees. And, for the blade tilt angle, $\sin(S^\circ) = \sin(90^\circ) = 1$, $\sin(180^\circ / N) = \sin(45^\circ) = 1 / \sqrt{2} \approx 0.707$, so $\cos(BT^\circ) = 1 / \sqrt{2}$, implying $BT^\circ = 45^\circ$. The miter gauge is set 90° degrees to the plane of the blade, and the blade is set 45° degrees to the saw table surface.

Example (the heptagon vase of Figure 4, having seven sides that slope seven degrees from the vertical): For this, $N = 7$ and $S^\circ = 83^\circ$ degrees. Then, $\cos(S^\circ) = \cos(83^\circ) \approx 0.1219$, $\tan(180^\circ / 7) \approx \tan(25.71^\circ) \approx 0.482$, so $\tan(MG^\circ) \approx 1 / (0.1219)(0.482) \approx 17.0196$, implying that $MG^\circ = 86.641^\circ$ degrees. For the blade tilt angle, $\sin(S^\circ) = \sin(83^\circ) \approx 0.9925$, $\sin(180^\circ / N) \approx \sin(25.71^\circ) \approx 0.434$, so $\cos(BT^\circ) \approx 0.4308$, implying $BT^\circ = 64.491^\circ$ degrees. The miter gauge is set to 86.641° degrees and the blade is tilted 64.491° degrees from the saw table surface.

Following is a Frink program to run on an Android platform (see Section 6. [Frink](#) for comments about the Frink programming language). The user inputs the number of sides and their slope, and the program returns the two setup angles for a table saw to cut compound miters.

CompoundMiter.Frink

```
// Calculates table saw settings for
//cutting sides of an multi-sided vessel
//having sides that slope.
// N   number of sides
// S   angle of the sides from horizontal (degrees)
// MG  miter gauge angle (degrees) from plane of blade
// BT  blade tilt angle (degrees) from saw surface
//Created By: D. L. Snyder 26 Feb 2011
//start.....
//user inputs
[N, S] = input["Vessel or Box Information",
    ["Number of sides",
    "Angle of sides from horizontal (degrees)"]]
//evaluate constants
S_deg = eval[S]
N_num = eval[N]
Corner_Angle = 360/N_num
a1 = sin[S_deg degrees]
a2 = sin[Corner_Angle/2 degrees]
b1 = cos[S_deg degrees]
b2 = tan[Corner_Angle/2 degrees]
//evaluate outputs
BT = arccos[a1*a2]
MG = arctan[1, b1*b2]
BTcomplement = pi/2-BT
BTsupplement = pi - BT
//display results
g = new graphics
g.font["SansSerif", 7 point]
g.backgroundColor[0,0,0.3]
g.color[0.5,0.5,0.5]
g.fillRectSize[-8,-10,19,1]
g.color[1,1,0]
g.text["SETUP ANGLES FOR COMPOUND MITERS",1,-10,"center","top"]
g.text["User Input Values",-9,-8.5,"left","center"]
    g.text[" Number of Sides = " + N_num, -9,-7.5, "left", "center"]
    g.text[" Slope of Sides (from horizontal) = " + eval[S] + " degrees",-9,-6.5,"left", "center"]
g.text["Output: Saw Setup Values", -9, -5, "left", "center"]
    g.text[" Miter Gauge Angle (from plane of blade) = " + format[MG, "degrees", 3],-9,-
4,"left", "center"]
    g.text[" Supplementary Miter-Gauge Angle(180 deg – MG) = " + format[MGsupplement, "de-
grees", 3],-9,-3.5,"left", "center"]
    g.text[" Blade Tilt Angle (from saw surface) BT = " + format[BT, "degrees", 3],-9,-
3,"left", "center"]
    g.text[" Complementary Blade-Tilt Angle (90 deg - BT) = " + format[BTcomplement, "degrees",
3],-9,-2,"left", "center"]
```



```

g.text[" Supplementary Blade-Tilt Angle (180 deg - BT) = " + format[BTsupplement, "degrees",
3],-9,-1,"left","center"]
g.show[]
//.....end

```

Following are screenshots of an Android Hero cellphone when this procedure is run using as input N = 7 sides, with each side tilted 83 degrees from horizontal (7 degrees from vertical).



Figure 26. Input Screen without (left) and with (right) user Input values

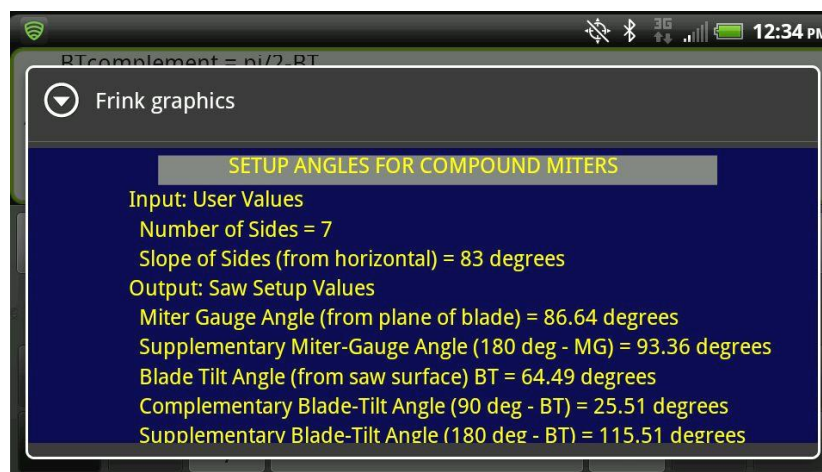


Figure 27. Output screen

4.1. Example: a small hexagonally shaped bowl



Figure 28. Hexagon shaped bowl

Here we outline the steps used to make the hexagonally shaped bowl in Figure 28. The sides of the bowl have a slope of 60° degrees to the base. The bottom and top edges of the sides have a 60° bevel; this permits the bottom of the bowl to sit flat on a table and the top edges to be parallel to the table top. The top of the sides of the bowl are scalloped to give the bowl a flower-like appearance. The bottom of the bowl is in the shape of a hexagon that fits into a dado that is cut into the sides at a 60° bevel angle. The material used is a length of cherry-wood that is about 3.0 inches wide, 0.5 inches thick, and long enough to obtain the eight or nine sides (to provide spare side pieces if needed.)

The first steps are to bevel the sides of the cherry-wood blank and cut in the dado for the bottom, as in Figure 29.

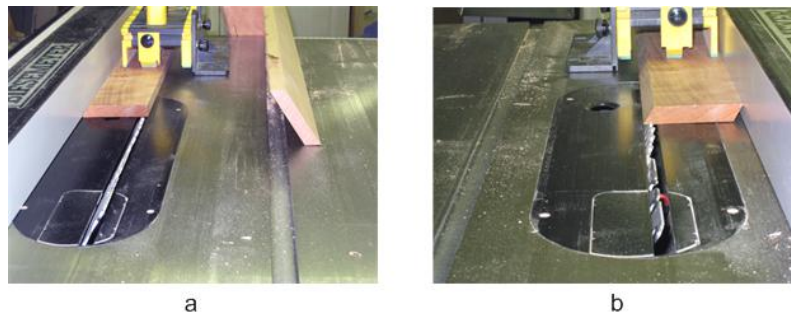


Figure 29. (a), bevel the edges of the blank; (b), add dado for bottom

There are 6 sides ($N = 6$) that slope 60° ($S^\circ = 60^\circ$) to the horizontal. Using either the equations for compound miters or the Frink program yields the setup angles in Figure 30 for cutting the sides on a table saw.

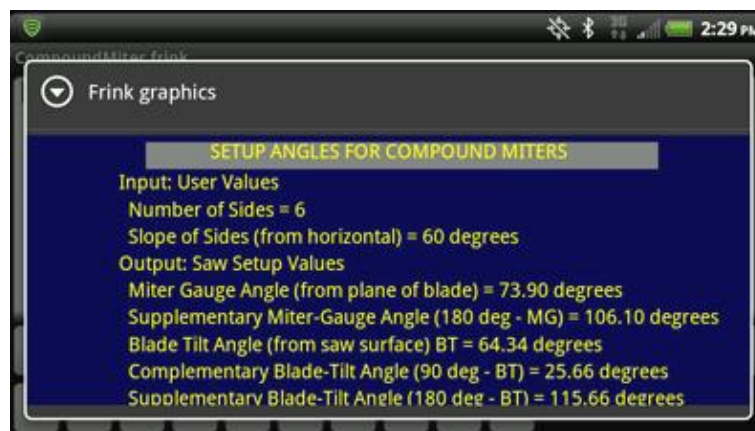


Figure 30. Compound miter angles for scalloped, hexagonal bowl

A protractor is used to set adjustable bevel gauges to the supplements to the miter-gauge and blade-tilt angles and then applied as in Figure 31 to set up the table saw.

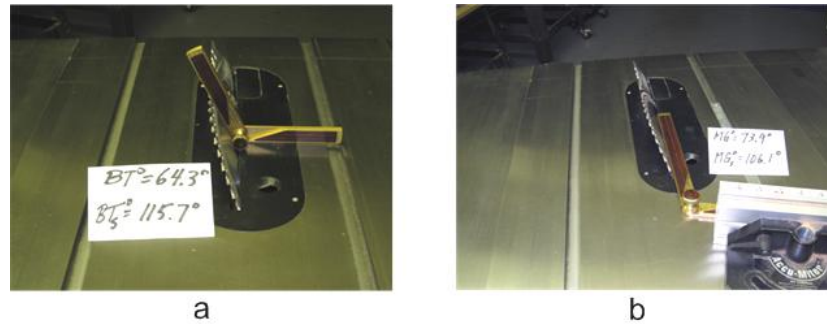


Figure 31. Setup table saw for compound miters: (a) blade-tilt angle set to 64.3° (saw's tilt indicator to 25.7°); miter-gauge angle set to 73.9°

The sides of the bowl are now cut from the blank using the three steps shown in Figure 32.

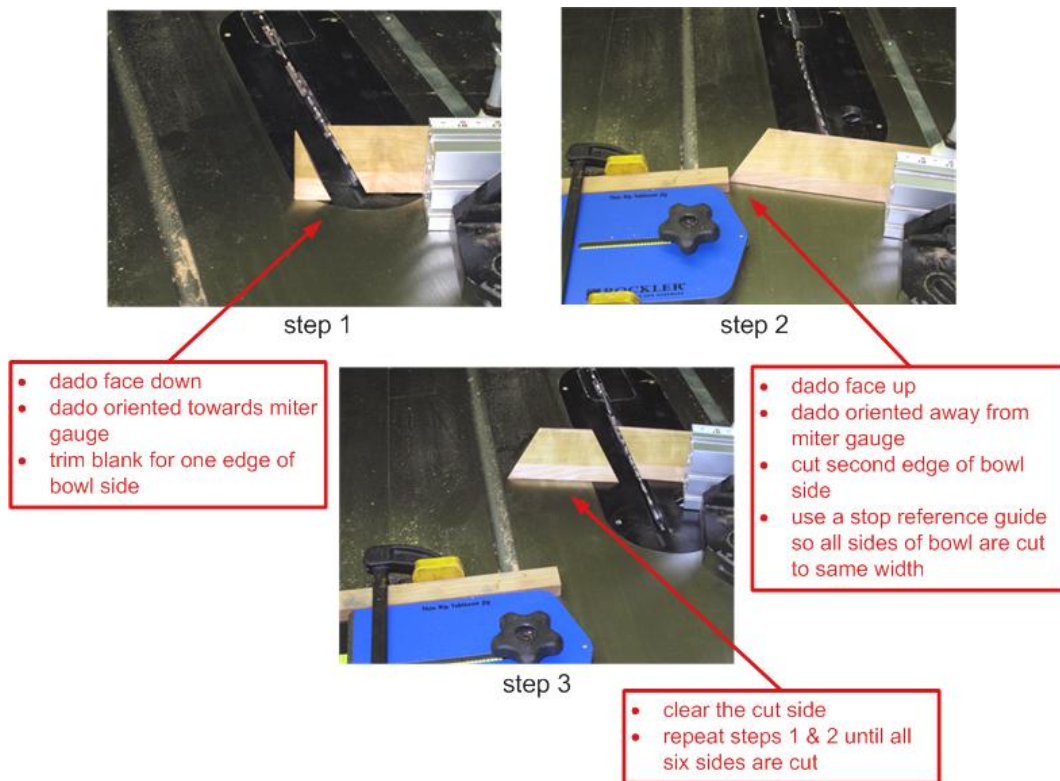
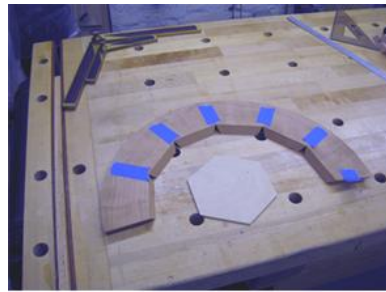
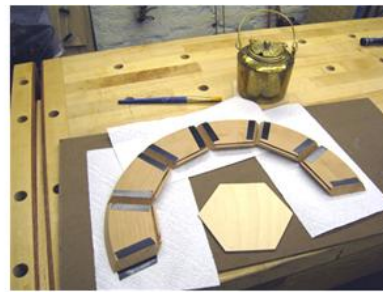


Figure 32. Steps used to cut the sides of the bowl

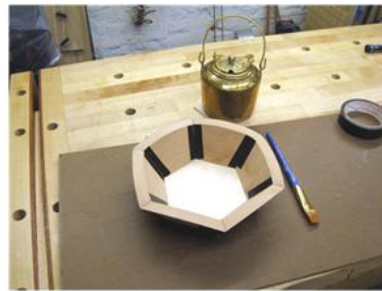
Tops of the sides are rounded to give the finished bowl a scalloped shape; this is done by drawing an arc at the top of each side and then using a belt sander (or other means) to remove material to that line. The sides are then glued together as shown in Figure 33.



tape the outside faces together



tape the inside faces to catch glue
squeeze out



apply glue and close up the sides with
bottom in dado

Figure 33. Glue the sides of the bowl together

Once the glue has dried, the bowl can be sanded and finish applied, with the result shown in Figure 28.

4.2. Example: a hexagonally shaped table lamp

The sides of the table lamp displayed in Figure 4 have a slope of 85° degrees to the horizontal. The bottom and top edges of the sides have a 5° degree bevel, permitting the top and bottom of the lamp to be joined to a flat, hexagonally shaped cap and base. The material used for the sides is padauk veneer laminated to $\frac{3}{4}$ in. MDF. This material begins as a blank that is about 10 in. wide and a length suitable to obtain the 48 sides (for eight lamps) plus some spares if needed. The cap and base are solid cherry wood.

There are 6 sides ($N = 6$) that slope 85° ($S^\circ = 85^\circ$) to the horizontal. Using either the equations for compound miters or the Frink program yields the setup angles in for cutting the sides on a table saw.

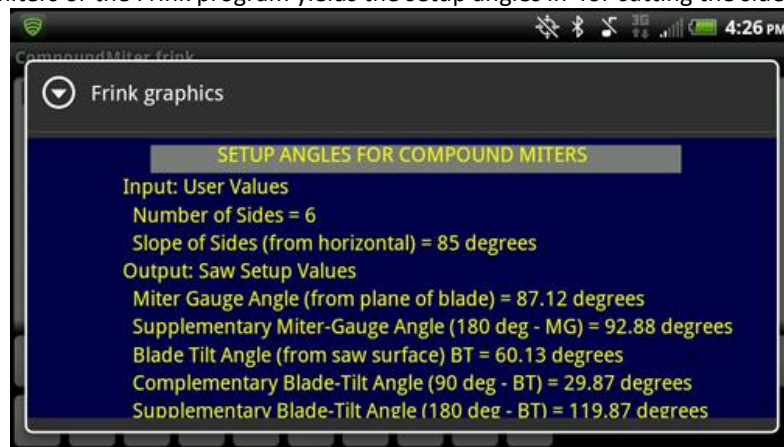


Figure 34. Table-saw setup angles for the table lamp

Beveling the two edges of the blank and cutting the sides proceeds in the same way as they did in Figure 29 and Figure 32 for the small bowl. One side of each of the eight lamps is excavated so that a brass drawer pull (item AD-4060 from Horton Brasses Inc.[see 7.12]) can be inset; this pull is decorative but also serves as a “touch” switch to operate the lamp. The sides after being glued together to form eight lamp bodies are displayed in Figure 35.



Figure 35. Eight hexagonally shaped lamp bodies

Eight hexagonal caps made of ½” thick cherry wood are made using the method described in Section 3.1. A center hole is drilled into each cap to accommodate lamp hardware. The capped lamp bodies are seen in Figure 36(a). The lamp bases are made in the form of a hexagonal

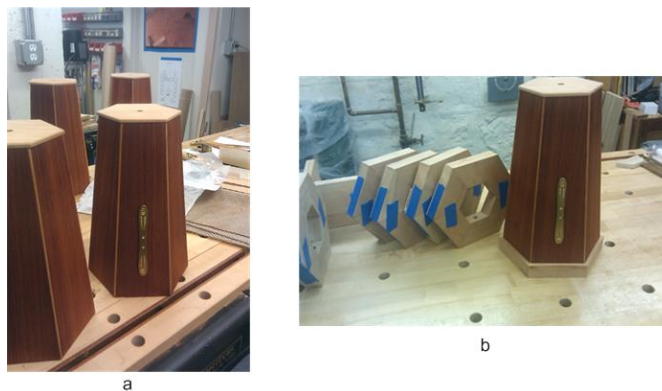


Figure 36. (a) capped lamp bodies; (b) lamp bases

frame, as seen in Figure 36(b). This requires simple miter cuts with the saw blade perpendicular to the saw surface and the miter gauge set to angle $A_3 = 60^\circ$ degrees. The lamp can now be finished, the hardware installed, and the electrical wiring completed. Included in the wiring is a switch [see 7.13], seen in Figure 37 , that only requires light touching to operate the lamp.

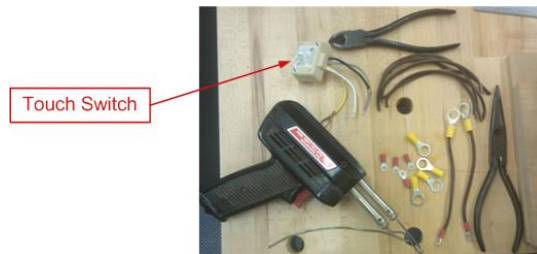


Figure 37. Electrical components include a touch switch

5. Polyhedra

Polygons are two-dimensional objects. Closed three-dimensional objects can be made from them by joining polygons along their edges. The objects obtained this way are called *polyhedra*. A cube obtained by connecting six squares together is an example of a polyhedron. Another example is the dodecahedron shown in Figure 38, which is built from twelve identical pentagons. Each pentagonal facet is 0.5 in. thick mahogany made using the pattern cutting method in Figure 22. Critical to constructing polyhedra are the bevel angles of its polygonal facets. This is discussed by D. Snyder [7.14], including the construction of the *polyhedral sundial* in Figure 39(c) made by placing sundials on the facets of the dodecahedron in Figure 38.



Figure 38. Dodecahedron made of mahogany



Figure 39. Polyhedral shaped objects:
(a) antique polyhedral box; (b) dodecahedral speaker system; (c) dodecahedral sundial

6. Frink

Frink is a computer-programming language that permits calculations, such as those for compound angles in Section 4, to be made on desktop computers and smart cell-phones, such as Android-based phones and iPhones. It and instructions for its use are available for free from various websites (see [7.15] and [7.16]). One way to implement Frink's use for creating procedures and applications (the short buzzword is "apps") for a cell-phone is to download Frink to both a desktop computer and the cell-phone to be used. Program the application on the desktop computer by using the programming mode in Frink or by loading into Frink a file, such as the compound-angle file listed in Section 4. Once the application works in the desktop environment, it can be transferred to the cell-phone via a direct or wireless connection to the desktop computer. After being loaded into the cellphone, the "app" can be used there via the Frink software on the cell-phone.

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