Laying Out Steve Latta's Ten-Leaf Lotus Flower

Using a Graphical Method for Constructing a Regular Decagon

Donald L. Snyder
April 2017

Introduction

Steve Latta is a prominent woodworking teacher at the *Thaddeus Stevens College of Technology*. He also teaches short courses at the *Marc Adams School of Woodworking* and other woodworking schools, and he gives many instructional presentations for woodworking clubs and conferences. One of his signature topics is decorative inlay. In an article published in *Fine Woodworking Magazine* in 2010, he describes in detail how he makes the ten-leaf lotus flower appearing in the wood panel he is holding in Figure 1.[1]

Figure 1. Steve Latta displaying his ten-leaf Lotus flower

Radii projecting from the center of the lotus flower towards the tips of the ten leaves are equally spaced in angle around a circle. Since there are ten leaves, the angle between any two adjacent radii is 36°. One approach for locating the 10 points for leaf tips is *trial-and-error* using a pair of dividers. This approach goes something like this: set the pins of the dividers to a candidate distance; place one pin of the dividers on the circle and locate the other pin on the circle; then, step the divider around the circle from point to point to see if a pin lands on the starting point after 10 steps; if not, adjust the distance between pins of the dividers by about one-tenth of the error and repeat; do this until the error is acceptably small, and mark the locations of the leaf tips on the circle.

[1] Steve Latta, "Dress up your work with creative stringing," *Fine Woodworking*, pp. 56-61, Nov/Dec 2010.

There is an alternative approach that removes the uncertainty that accompanies this trial-and-error method and results directly in locating the ten leaf tips at the requisite 36° intervals. It is a graphical method requiring only a compass and a ruler. It can be precise if the nine steps to be described are performed carefully.

A graphical approach for locating leaf tips

A regular decagon is a ten-sided polygon in which all sides are of equal length, and all ten interior angles (the angles subtended by the sides at the center of the decagon) equal 36°. The ten tips of the leaves in Latta's lotus-flower design are at the vertices of a regular decagon. Following are steps for drawing the largest decagon that fits within a circle of a specified (or even an unknown) radius and thereby locating the leaf tips.

Step 1. Draw a circle of radius *R*. *Comment*: the radii used for the three circles in Latta's lotus-flower design (shown on p. 59 of his article) are 7/8, 1¾, and 2½ inches.

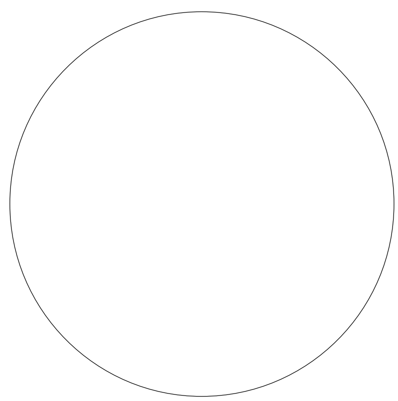

Figure 2. Circle of some radius R

Step2. Locate the center of the circle. One method is to draw two chords, and then draw the perpendicular bisectors of the chords.[2] These bisectors cross at the center of the circle, as seen in Figure 3.

[2] The perpendicular bisector of a line is obtained by drawing two circles centered at the ends of the line and having a common radius that is greater than half the length of the line. The perpendicular bisector is the line passing through the points of intersection of the two circles.

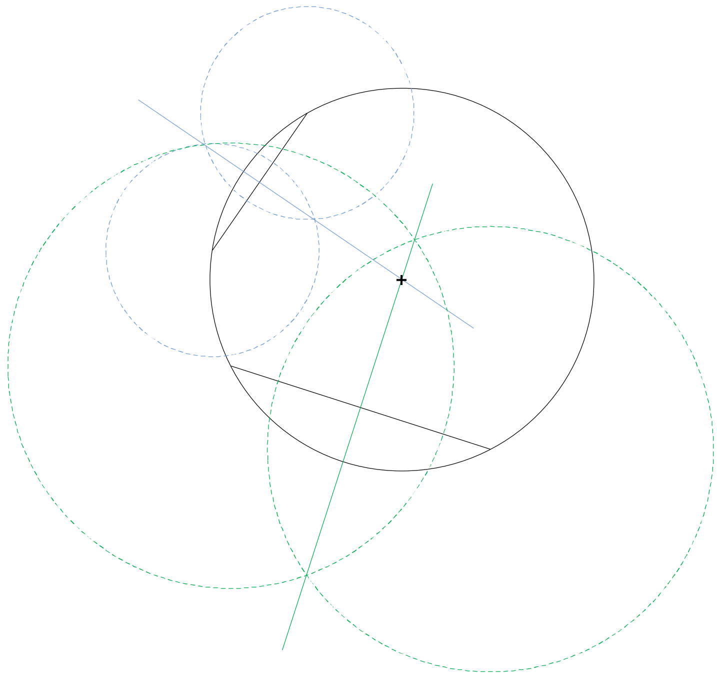

Figure 3

Step 3. Draw a line through the center of the circle, as in Figure 4. This line can be drawn at any desired angle to achieve a preferred orientation of the decagon and hence of the lotus flower.

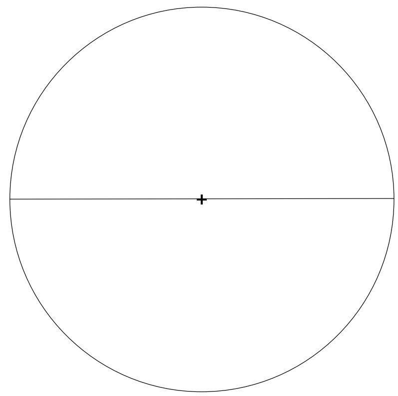

Figure 4

Step 4. Draw a perpendicular bisector of that line. This can be accomplished with the compass-and-ruler method used in Step 2. The result is as displayed in Figure 5.

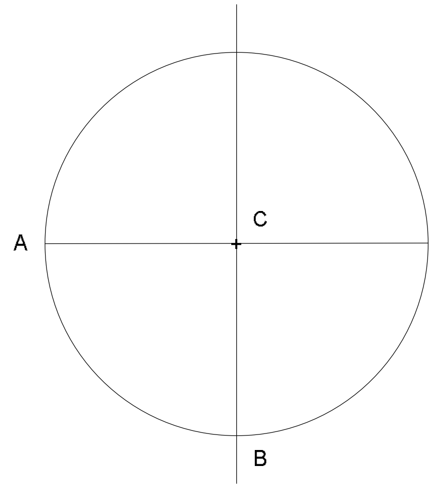

Figure 5. Perpendicular lines through the center of a circle

Step 5. Determine the center (D) of the line segment A-C in Figure 6. This can be accomplished using the compass-and-ruler method in Step 2.

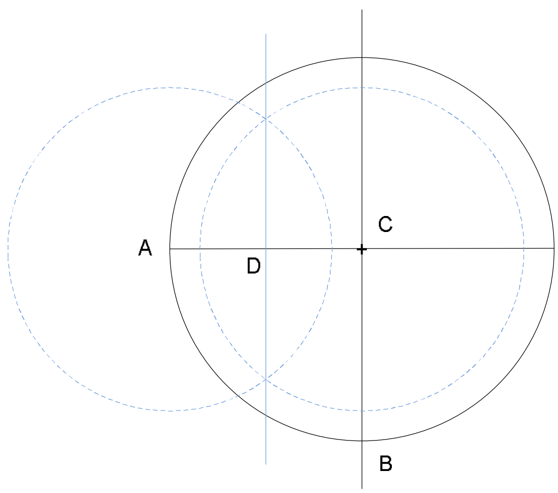

Figure 6. Perpendicular bisector of a radial line

Step 6. Draw a circle of radius *R/2* centered at D, as in Figure 7.

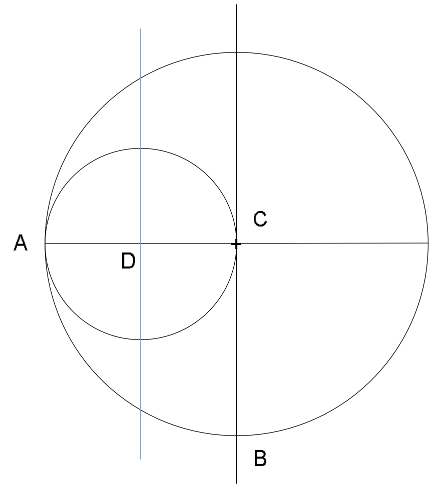

Figure 7. Interior circle of half radius R

<u>Step 7.</u> Draw a line from B to D, and note the point E where this line intersects the circle of radius *R/2*, as in Figure 8.

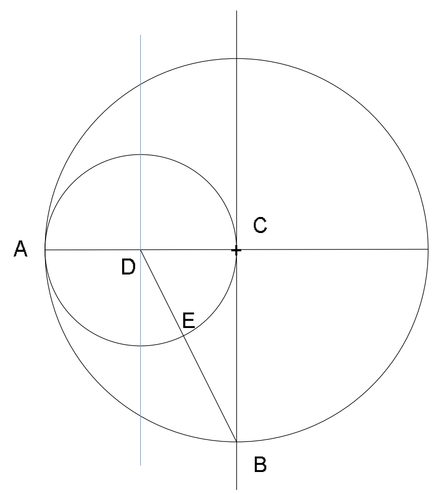

Figure 8

<u>Step 8.</u> Using a compass, draw an arc of radius B-E centered at B, and note the point F where this intersects the original circle of radius *R* as in Figure 9.

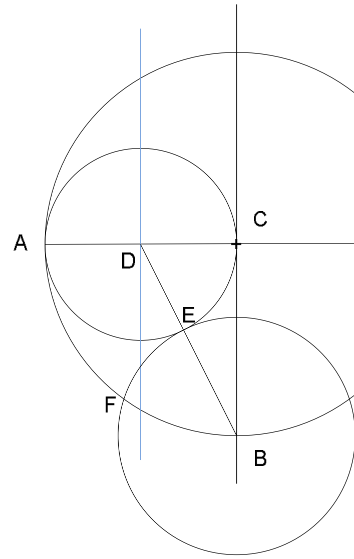

Figure 9

Step 9. The points F and B are adjacent vertices of the largest decagon inscribed within the original circle of radius *R*. Using the same compass setting as in Step 8, use the compass to step around the circle of radius *R* marking the remaining eight vertices of the decagon.

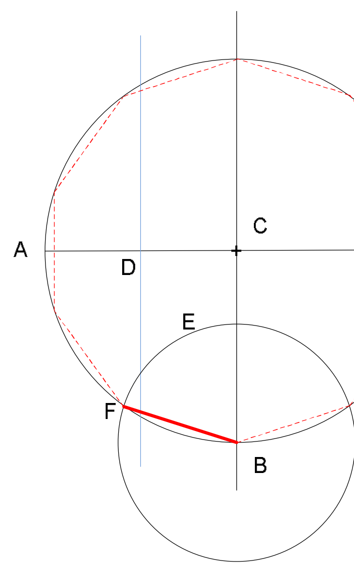

Figure 10

The angle B-C-F is 36°. Here is one way to demonstrate that this is so. Note from Figure 9 that the length of the line segment C-B is *R* and that of C-D is *R/2*. The Pythagorean Theorem implies that the length *L* of the line segment B-D is

$$L = \sqrt{R^2 + (R/2)^2} = R\sqrt{5}/2 .$$

Thus, the length of the line segment B-E is

$$L-\frac{1}{2}R=\frac{1}{2}\left(\sqrt{5}-1\right)R=R/\phi \ ,$$

where $\phi=\left(\sqrt{5}+1\right)/2$ is the Golden Ratio.[3] Because of the arc construction in Step 8, this is also the length of B-F and, therefore, also of each side of the decagon. From the cosine law,[4] the angle B-C-F, call it θ, satisfies

$$\left[\frac{1}{2}\left(\sqrt{5}-1\right)R\right]^2=R^2+R^2-2R^2\cos\theta \ .$$

Thus,

$$\cos\theta=1-\frac{1}{8}\left(\sqrt{5}-1\right)^2 \ ,$$

and

$$\theta=\arccos\left[1-\frac{1}{8}\left(\sqrt{5}-1\right)^2\right]=36^\circ.$$

Once the locations of the ten leaf tips are identified, following the rest of Latta's instructions will result in constructing his lotus flower.

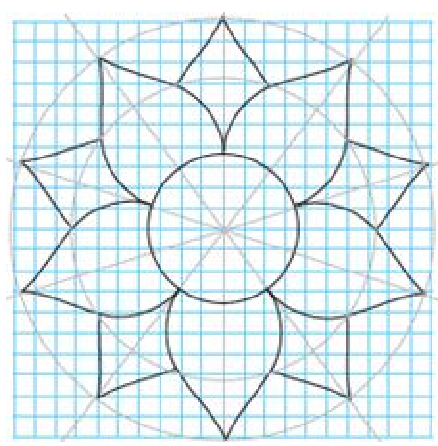

Figure 11. Steve Latta's lotus flower (from: Fine Woodworking, No. 215, p. 59, Dec. 2010)

As a final comment, it is seen that five of the ten leaves are partially hidden behind the other five and that they emerge into view at points along radial lines that lie midway between radial lines to the points of the ten leaves. These bisecting radial lines can be constructed easily by using a compass and ruler to bisect the angles between adjacent leaf tips.

---

[3] The occurrence of the Golden Ratio is interesting but not important for this discussion. See: M. Livio, *Golden Ratio*, p. 80, Broadway Books (Random House), 2002.

[4] Consider a triangle having side lengths *a*, *b* and *c*. The angle ω between sides *a* and *b* satisfies $c^2=a^2+b^2-2ab\cos\omega$. This is called the *cosine law* in elementary trigonometry.